Application Demonstration

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FRACTURE PROPAGATION IN THE INTERNAL PRESSURIZED VESSEL

APPROVED
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** *Abstract* :
-Calculating the maximum value Pmax that causes the fracture propagation while applying the internal static pressure .
-Evaluating the time-limit for applying the continous / discrete form of internal dynamic pressure for the round metal vessel under conditions given by series expansion and curve -fitting method .
** Subjects: Fracture Mechanics , The stress intensity factor KI .

Introduction

This worksheet demonstrates Maple's capabilities in estimating the maximum pressure and the time-limit for applying static or dynamic pressure on the surface of a round metal vessel which has an internal semi-elliptical surface crack .

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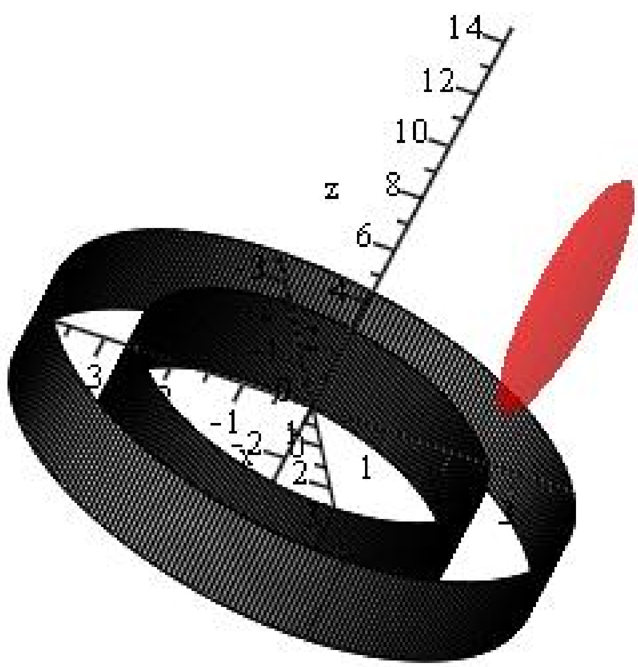
1. Modeling the problem

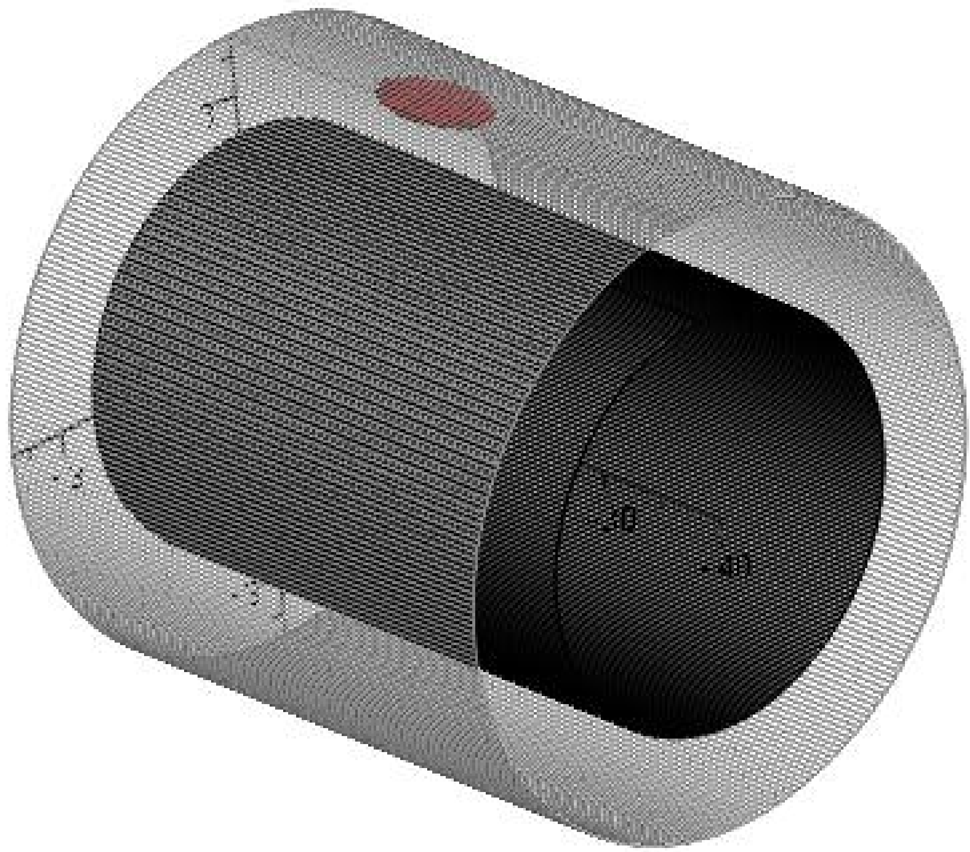
We consider the round metal vessel with parameters given :

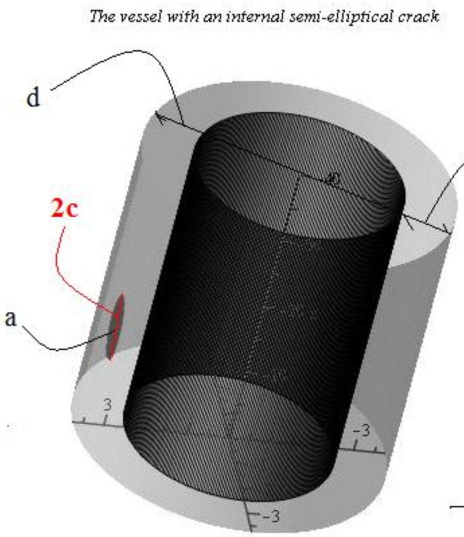
a : one-half crack depth (mm) ; B : thickness of vessel cm) ; c : one-half crack length (mm) ; d : internal diameter(m) ; P : internal pressure (MPa) .

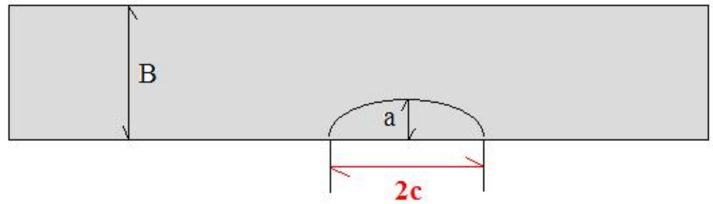
Modeling the sizes of the vessel **and** *crack*

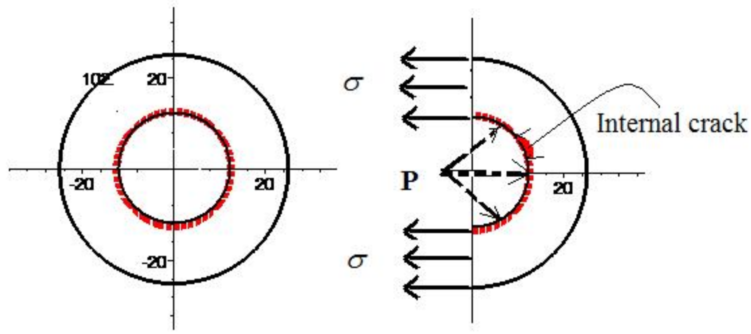


The vessel with an internal semi-elliptical crack









2. Technical basis and calculation formulas

Depending on the principle of superposition , the total stress intensity factor will be determined by :

$$K_{I\,total} = K_{I\,Hoop} + K_{I\,Pressure}$$

After calculating , $K_{I\,total}$ can be rewritten in the form : $K_{I\,total} := \alpha \cdot P_{pressure} \cdot \sqrt{a\pi}$

$$\left(\alpha \; : correction\,factor = \frac{1.12}{\sqrt{Q_{shape}}} \left(\frac{d_{diameter}}{2 \cdot B_{thickness}} + 1 \right) \right.$$

Use the formulas of hoop stress : $\sigma_{hoop} = \frac{1}{2} \frac{P_{press}\, d_{diameter}}{B_{thickness}}$ and

the shape factor $Q_{shape} = \frac{1}{4} \pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2} \right)^2 - \frac{7}{33} \frac{\sigma^2}{\sigma_{ys}^2}$, we find the value of correction factor

α , consequently it gives the total stress intensity $K_{I\,total}$. Because the critical crack length is

defined by : $a_{crack} = \frac{1}{\pi} \cdot \left(\frac{K_{Ic}}{\alpha_{correction} \cdot \sigma} \right)^2$, we may conclude that the maximum crack length has a

interactive relation with the stress intensity factor $K_{I\,total}$.

Following is the table which shows temperature mechanical properties for some materials used in engineering design .

Table 1.

Material	σ_{ys}	σ_{ts}	K_{Ic}	E
AerMet 100	1724	1965	126	207
$Ti-6\,Al-4\,V$ 1	869	958	87	117
$Ti-6\,Al-4\,V$ 2	1007	1034	40	130
AISI 4340 1	1089	1097	110	207
AISI 4340 2	1476	1896	81	207
AISI 4147	945	1062	120	207
INCONEL	1172	1404	96	-
18 *Ni* (250)	1290	1345	176	-
2014 -*T651*	455	-	24	-
2024 -*T3*	345	-	44	-
7075 -*T6*	572	641	24	72
$Ni_{49}\,Fe_{29}\,P_{14}\,B_6\,\mathrm{Si}_2$	800	800	12	-
SIC	460	460	3.7	72

3. Construct algorithms for applications

Here the author will present two procedures that are used to calculate the maximum pressure (Pmax) causing the fracture propagation and estimate the limitation of time while applying the continous or discrete internal pressure functions for the round metal vessel under the given constraints .

3.1 Calculating the maximum pressure :

The matrix A below is used to define the mechanical properties of each type of metal which is required for engineering design .

$pressure_{Max} := \mathbf{proc}(a1, B1, c1, d1, P1, type1, k)$

the data of problem will be entered into the procedure with the instructions below
a1 : one-half crack depth (mm) ,
B1 : thickness of vessel cm) ,
c1 : one-half crack length (mm) ,
d1 : internal diameter(m) ,
P1 : internal pressure (MPa) ,
type1 : type of material (inserted in the form of an element of the matrix A i.e A [k][i]) ,
k : the kth row (see table 1.) .

> $restart; A := Matrix([[material, \sigma_{ys}, \sigma_{ts}, K_{IC}, E], [AerMet100, 1724, 1965, 126, 207], [Ti6Al4V1, 869, 958, 87, 117], [Ti6Al4V2, 1007, 1034, 40, 130], [AISI43401, 1089, 1097, 110, 207], [AISI43402, 1476, 1896, 81, 207], [AISI4147, 945, 1062, 120, 207], [INCONEL, 1172, 1404, 96, NA], [18\,Ni250, 1290, 1345, 176, NA], [2014\,T651, 455, NA, 24, NA], [2024\,T3, 345, NA, 44, NA], [7075\,T6, 572, 641, 24, 72], [Ni49Fe29P14B6Si2, 800, 800, 12, NA], [SIC, 460, 460, 3.7, 72]])$

$$A := \begin{bmatrix} \text{14 x 5 Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \tag{1}$$

> $pressure_{Max}$:=**proc**$(a1, B1, c1, d1, P1, type1, k)$ **local** $i, s, anew, cnew, Bnew$, sigma, $sigma1, alpha1, alphax, P, Q, Q1, K, KI1, KIC$; $with(plots, implicitplot)$:
$print($" Start the program "$)$; $print\left(\text{" The parameters are given : "}, a = a1, B = B1, 2\,c = 2\cdot c1, d = d1, P = P1, type = type1, \frac{a}{2\cdot c} = evalf\left(\frac{a1}{2\cdot c1}, 3\right), row = k\right)$;
$print($" Convert the units into meter : "$)$; $anew := \frac{a1}{1000}$; $Bnew := \frac{B1}{100}$; $cnew := \frac{c1}{1000}$; $print(a = anew, B = Bnew, c = cnew, d = d1)$; $K_I :=$ alpha$\cdot P\cdot\sqrt{\pi\cdot a}$;
$print\left(\text{" (1) The total stress intensity factor KI "} = \text{alpha}\cdot P_{pressure}\cdot\sqrt{\pi\cdot a}\right)$; ; $alphax := \frac{1.12}{\sqrt{Q_{shape}}}\cdot\left(\frac{d_{diameter}}{2\cdot B_{thickness}} + 1\right)$; $print\left(\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12}{\sqrt{Q_{shape}}}\cdot\left(\frac{d}{2\cdot B} + 1\right)\right)$; $Q_{shape} := \frac{\pi^2}{4}\cdot\left(\frac{3}{4} + \left(\frac{a}{2\cdot c}\right)^2\right)^2 - \frac{7}{33}\cdot\left(\frac{\sigma}{\sigma_{ys}}\right)^2$; sigma $:= \frac{P_{pressure}\cdot d_{diameter}}{2\cdot B_{thickness}}$;
$print\left(\text{" (3) The hoop stress } \sigma \text{ "} = \frac{P_{pressure}\cdot d_{diameter}}{2\cdot B_{thickness}}\right)$; ;
$print($" (4) The shape factor Qshape " $= Q_{shape})$; $printf($"\n%s",
" Read the data from the temperature mechanical properties table"$)$; $printf($"\n%s", " "$)$;;
$print(TYPE)$;**for** i **from** 1 **to** 5 **do** $print(s[i] = A[k][i])$; $s[i] := A[k][i]$; **od**; ;
$print\left(Material = s[1], \Sigma_{ys} = s[2], \Sigma_{ts} = s[3], KIc = s[4], E = s[5]\right)$; ; ; $printf($"\n%s",
" Calculating the hoop stress sigma , the shape factor Q , correction factor alpha and the total stress intensity factor KI "$)$; $printf($"\n%s", " "$)$; $sigma1 := evalf\left(\frac{P1\cdot d1}{2\cdot Bnew}, 5\right)$;
$print($" From (3) "$)$; $print\left(\text{" (3) The hoop stress } \sigma \text{ "} = \frac{P_{pressure}\cdot d_{diameter}}{2\cdot B_{thickness}}\right)$;
$print($" σ " $= sigma1)$;; $printf($"\n%s", " "$)$; $print($" From (4) "$)$;
$print($" (4) The shape factor Q " $= Q_{shape})$; $\sigma_{ys} := s[2]$; $Q1 := \frac{\pi^2}{4}\cdot\left(\frac{3}{4}\right.$

$+ \left(\frac{anew}{2\cdot cnew}\right)^2\Big)^2 - \frac{7}{33}\cdot\left(\frac{\sigma 1}{\sigma_{ys}}\right)^2$; $Q1 := evalf(Q1, 3)$; ; $print($" Qshape " $=Q1)$; ;

$printf$("\n%s", " "); $print$(" From (2) "); $print\left(\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12}{\sqrt{Q_{shape}}}\cdot\left(\frac{d_{diameter}}{2\cdot B_{thickness}} + 1\right)\right)$; $alpha1 := \frac{1.12}{\sqrt{Q1}}\cdot\left(\frac{d1}{2\cdot Bnew} + 1\right)$; $alpha1 := evalf(alpha1, 3)$;

$print$(" α " $= alpha1$); ; $printf$("\n%s", " "); $print$(" From (1) ");

$print\left(\text{" (1) The total stress intensity KI "} = \text{alpha}\cdot P_{pressure}\cdot\sqrt{\pi\cdot a}\right)$; $KI1 := alpha1\cdot P1 \cdot\sqrt{\pi\cdot anew}$; $print(KI = KI1)$; $KI1 := evalf(KI1, 4)$; $print(KI = KI1)$; ; $printf$("\n%s", " "); $printf$("\n%s", " 1/. Compare the total stress intensity factor KI and KIc "); ;

$printf$("\n%s", " "); $KIC := s[4]$; $print(KIc = KIC)$;**if** $KI1 < KIC$

then $print$(" The fracture will not occur / the crack propagation is stable , because KI < KIc "); ; $printf$("\n%s", " "); ; $printf$("\n%s", " "); $printf$("\n%s", " 2/. Determine the maximum pressure Pmax that causes the crack propagation "); ;

$printf$("\n%s", " "); $print$(" From (1) "); $print\left(\text{" (1) The critical stress intensity KIc "} = \alpha[c]\cdot P_{Max}\cdot\sqrt{\pi\cdot a}\right)$; $P_{MAX} := \frac{KIC}{\alpha 1\cdot\sqrt{\pi\cdot anew}}$; $P_{MAX} := evalf(P_{MAX}, 5)$; $print(Pmax = P_{MAX})$; ; $printf$("\n%s", " "); $printf$("\n%s", " 3/. Consider the relation between Q and α "); ; $printf$("\n%s", " ");

$print$(" From (2) "); $print$(" Substitute x for alpha and y for Qshape into (2) ");

$print(\text{alpha} = alphax)$; $x = evalf\left(\frac{1.12}{\sqrt{y}}\cdot\left(\frac{d1}{2\cdot Bnew} + 1\right), 4\right)$; $print(\%, plot1)$; $y = x\cdot P1 \cdot\sqrt{\pi\cdot anew}$;; $print(\%, plot2)$;; $printf$("\n%s", " ");; $printf$("\n%s", " ");

$print$(" Graphical relations between correction and shape factor (plot1) _ correction and stress intensity factor KI (plot2) ");; $print$(" End of program "); ; $printf$("\n%s", " ");

$plot\left(\left[evalf\left(\frac{1.12}{\sqrt{x}}\cdot\left(\frac{d1}{2\cdot Bnew} + 1\right), 4\right), x\cdot P1\cdot\sqrt{\pi\cdot anew}\right], x = 0..4, y = 0..20, axes = boxed, labels = [\text{`Q , KI`}, \text{`alpha`}], color = [blue, red], legend = [plot1, plot2], thickness = [2, 1]\right)$;

else $print$(" Warning ! The fracture will occur / the crack propagation is unstable , because KI >= KIc ");; $print$(" End of program ");; **fi**; **end**:

Example 3.1.1

Consider a round vessel with technical data below :
one-half crack depth a = 5.5mm ; thickness of vessel B = 20cm ; one-half crack length c = 4.5mm ; internal diameter d = 0.5m ; internal pressure P = 362 MPa ; type : AISI 4340 2 i.e A[6][1]
Calculate :
a. The stress intensity factor KI .

b.The maximum static pressure that would cause the facture propagation .
Compare the total stress intensity factor KI and KIc (see table 1.) and give your conclusion .

Solution :
Run the procedure *

> $pressure_{Max}(5.5, 20, 4.5, 0.5, 362, A[6][1], 6);$

" Start the program "

" The parameters are given : ", $a = 5.5, B = 20, 2\,c = 9.0, d = 0.5, P = 362, type = AISI43402,$
$$\frac{1}{2}\,\frac{a}{c} = 0.610,\ row = 6$$

" Convert the units into meter : "

$$a = 0.005500000000,\ B = \frac{1}{5},\ c = 0.004500000000,\ d = 0.5$$

$$\text{" (1) The total stress intensity factor KI "} = \alpha\,P_{pressure}\sqrt{\pi\,a}$$

$$\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12\left(\frac{1}{2}\,\frac{d}{B} + 1\right)}{\sqrt{Q_{shape}}}$$

$$\text{" (3) The hoop stress } \sigma \text{ "} = \frac{1}{2}\,\frac{P_{pressure}\,d_{diameter}}{B_{thickness}}$$

$$\text{" (4) The shape factor Qshape "} = \frac{1}{4}\,\pi^2\left(\frac{3}{4} + \frac{1}{4}\,\frac{a^2}{c^2}\right)^2 - \frac{7}{33}\,\frac{\sigma^2}{\sigma_{ys}^2}$$

```
Read the data from the temperature mechanical properties table
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$$TYPE$$
$$s_1 = AISI43402$$
$$s_2 = 1476$$
$$s_3 = 1896$$
$$s_4 = 81$$
$$s_5 = 207$$
$$Material = AISI43402,\ \Sigma_{ys} = 1476,\ \Sigma_{ts} = 1896,\ KIc = 81,\ E = 207$$

```
 Calculating the hoop stress  sigma , the shape factor Q ,
correction factor alpha  and the total stress intensity factor
KI
```

" From (3) "

$$"\ (3)\ \text{The hoop stress}\quad \sigma\ " = \frac{1}{2}\,\frac{P_{pressure}\,d_{diameter}}{B_{thickness}}$$

$$"\ \sigma\ " = 452.50$$

$$"\ \text{From (4)}\ "$$

$$"\ (4)\ \text{The shape factor}\quad Q\ " = \frac{1}{4}\,\pi^2\left(\frac{3}{4}+\frac{1}{4}\,\frac{a^2}{c^2}\right)^2 - \frac{7}{33}\,\frac{\sigma^2}{\sigma_{ys}^2}$$

$$"\ \text{Qshape}\ " = 3.10$$

$$"\ \text{From (2)}\ "$$

$$"(2)\ \text{Correction factor}\quad \alpha\ " = \frac{73.92\left(\frac{1}{2}\,\frac{d_{diameter}}{B_{thickness}}+1\right)}{\sqrt{1089\,\pi^2\left(\frac{3}{4}+\frac{1}{4}\,\frac{a^2}{c^2}\right)^2 - \frac{924\,\sigma^2}{\sigma_{ys}^2}}}$$

$$"\ \alpha\ " = 1.43$$

$$"\ \text{From (1)}\ "$$

$$"\ (1)\ \text{The total stress intensity}\quad KI\ " = \alpha\,P_{pressure}\sqrt{\pi\,a}$$

$$KI = 38.39069309\,\sqrt{\pi}$$

$$KI = 68.07$$

1/. Compare the total stress intensity factor KI and KIc

$$KIc = 81$$

" The fracture will not occur / the crack propagation is stable , because KI $<$ KIc "

2/. Determine the maximum pressure Pmax that causes the crack propagation

$$"\ \text{From (1)}\ "$$

$$"\ (1)\ \text{The critical stress intensity}\quad KIc\ " = \alpha_c\,P_{Max}\sqrt{\pi\,a}$$

$$Pmax = 430.90$$

```
3/. Consider the relation between Q and &alpha;
```

" From (2) "

" Substitute x for alpha and y for Qshape into (2) "

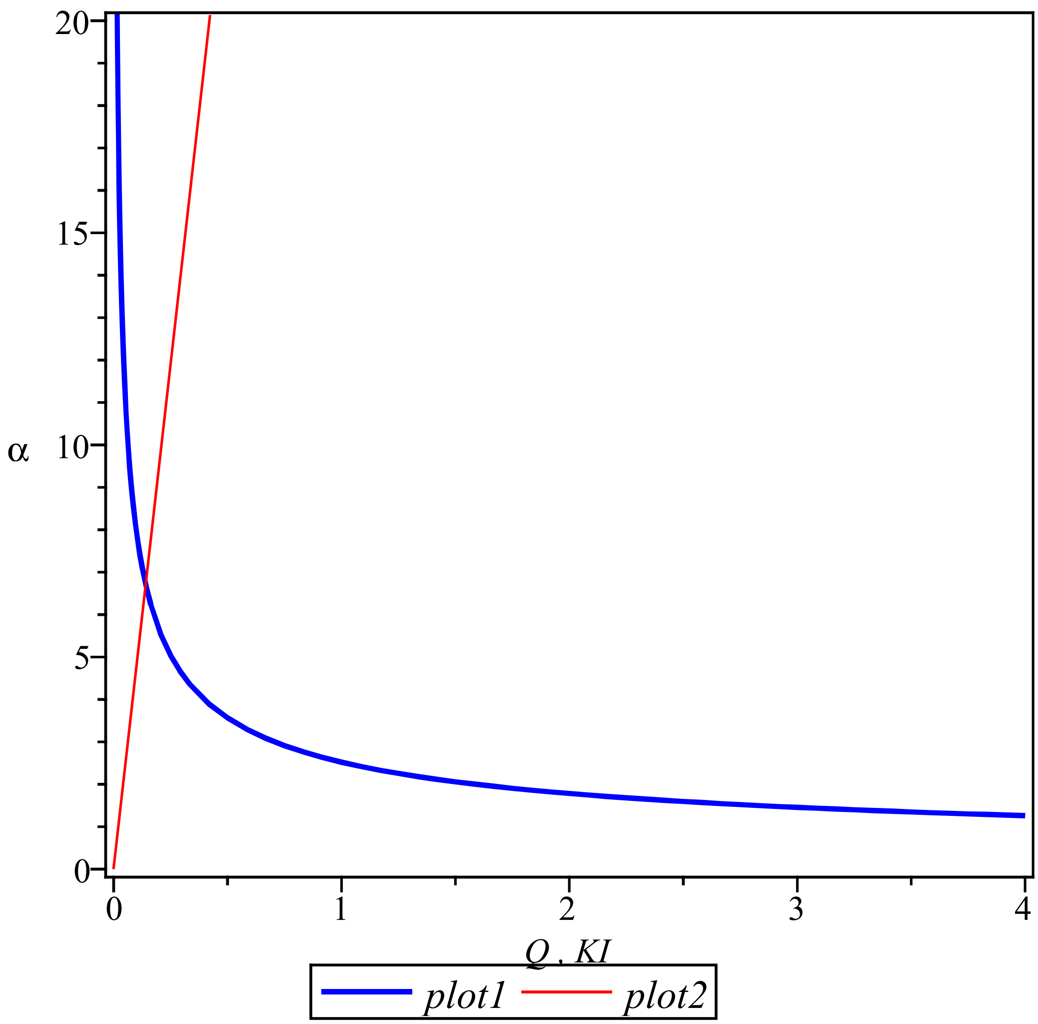
$$\alpha = \frac{1.12\left(\frac{1}{2}\,\frac{d_{diameter}}{B_{thickness}} + 1\right)}{\sqrt{Q_{shape}}}$$

$$x = \frac{2.520}{\sqrt{y}}, plot1$$

$$y = 26.84663852\,x\,\sqrt{\pi}, plot2$$

" Graphical relations between correction and shape factor (plot1) _ correction and stress intensity factor KI (plot2) "

" End of program "



Example 3.1.2

Consider a round vessel with technical data below :
one-half crack depth a = 4.5mm ; thickness of vessel B = 20cm ; one-half crack length c = 6mm ; internal diameter d = 0.8m ; internal pressure P = 560 MPa ; type : AISI 4340 1 i.e A[5][1]
Calculate :
a. The stress intensity factor KI .
b.The maximum static pressure that would cause the facture propagation .
Compare the total stress intensity factor KI and KIc (see table 1.) and give your conclusion .

Solution :
Run the procedure $pressure_{Max}(, , ,);$

```
> pressure_Max(4.5, 20, 6, 0.8, 560, A[5][1], 5);
```

" Start the program "

" The parameters are given : ", $a = 4.5$, $B = 20$, $2\ c = 12$, $d = 0.8$, $P = 560$, $type = AISI43401$, $\frac{1}{2}\ \frac{a}{c} = 0.375$, $row = 5$

$$" \text{ Convert the units into meter : } "$$

$$a = 0.004500000000, B = \frac{1}{5}, c = \frac{3}{500}, d = 0.8$$

$$" \text{(1) The total stress intensity factor KI} " = \alpha P_{pressure} \sqrt{\pi a}$$

$$"\text{(2) Correction factor } \alpha " = \frac{1.12 \left(\frac{1}{2} \frac{d}{B} + 1 \right)}{\sqrt{Q_{shape}}}$$

$$" \text{(3) The hoop stress } \sigma " = \frac{1}{2} \frac{P_{pressure} d_{diameter}}{B_{thickness}}$$

$$" \text{(4) The shape factor Qshape } " = \frac{1}{4} \pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2} \right)^2 - \frac{7}{33} \frac{\sigma^2}{\sigma_{ys}^2}$$

```
Read the data from the temperature mechanical properties table
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$$TYPE$$

$$s_1 = AISI43401$$

$$s_2 = 1089$$

$$s_3 = 1097$$

$$s_4 = 110$$

$$s_5 = 207$$

$$Material = AISI43401, \Sigma_{ys} = 1089, \Sigma_{ts} = 1097, KIc = 110, E = 207$$

```
Calculating the hoop stress  sigma , the shape factor Q ,
correction factor alpha  and the total stress intensity factor
KI
```

$$" \text{ From (3) } "$$

$$" \text{(3) The hoop stress } \sigma " = \frac{1}{2} \frac{P_{pressure} d_{diameter}}{B_{thickness}}$$

$$" \sigma " = 1120.0$$

$$" \text{ From (4) } "$$

$$" \text{(4) The shape factor Q } " = \frac{1}{4} \pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2} \right)^2 - \frac{7}{33} \frac{\sigma^2}{\sigma_{ys}^2}$$

$$" \text{ Qshape } " = 1.73$$

" From (2) "

$$\text{"(2) Correction factor } \alpha \text{ "} = \frac{73.92 \left(\frac{1}{2} \frac{d_{diameter}}{B_{thickness}} + 1 \right)}{\sqrt{1089\,\pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2} \right)^2 - \frac{924\,\sigma^2}{\sigma_{ys}^2}}}$$

$$\text{" } \alpha \text{ "} = 2.55$$

" From (1) "

$$\text{" (1) The total stress intensity KI "} = \alpha\, P_{pressure} \sqrt{\pi\, a}$$

$$KI = 95.79315215 \sqrt{\pi}$$

$$KI = 169.8$$

```
1/. Compare the  total stress intensity factor KI  and KIc
```

$$KIc = 110$$

" Warning ! The fracture will occur / the crack propagation is unstable , because KI >= KIc "

" End of program " **(2)**

3.2 Estimating the interval of time while applying the internal dynamic pressure

For the next procedure $pressure_{Estm} :=$ **proc**$(a1, B1, c1, d1, P1, type1, k, tlim, t0, y0, y1)$

we enter the data of the problem with following instructions

a1 : one-half crack depth (mm) ,
B1 : thickness of vessel cm) ,
c1 : one-half crack length (mm) ,
d1 : internal diameter(m) ,
P1 : internal pressure function (continous /discrete) (MPa) ,
type1 : type of material (inserted in the form of an element of the matrix A i.e A [k][i]) ,
k : the kth row (see table 1.) ,
tlim : the upper bound of interval (0 , tlim) that is a constraint of time used to solve the inequality KI < KIc ,
t0, y0 , y1 : the minimum and maximum values of t and y used to plot the graph of funtions KI and the polynomial KITaylor .

> $pressure_{Estm} :=$ **proc**$(a1, B1, c1, d1, P1, type1, k, tlim, t0, y0, y1)$ **global** $P2, KIC, KI1, KI2$;
local $st, st2, ineq, ineq2, i, s, anew, cnew, Bnew,$ sigma , $sigma1$, $alpha1$, $alphax, Q, P,$

$Q1, K$; $with(plots, implicitplot)$: $print($" Start the program " $)$;

$print\Big($" Parameters of the metal vessel and internal pressure P : ", $a = a1, B = B1, 2\ c = 2 \cdot c1, d = d1, P = P1, type = type1, \frac{a}{2 \cdot c} = evalf\left(\frac{a1}{2 \cdot c1}, 3\right), row = k\Big)$;

$print($" Convert the units into meter : "$)$; $anew := \frac{a1}{1000}$; $Bnew := \frac{B1}{100}$; $cnew := \frac{c1}{1000}$; $print(a = anew, B = Bnew, c = cnew, d = d1)$; $P2 := P1$; $K_I := \text{alpha} \cdot P \cdot \sqrt{\pi \cdot a}$;

$print\Big($ " (1) The total stress intensity factor KI " $= \text{alpha} \cdot P_{pressure} \cdot \sqrt{\pi \cdot a}\Big)$; ; $alphax := \frac{1.12}{\sqrt{Q_{shape}}} \cdot \left(\frac{d_{diameter}}{2 \cdot B_{thickness}} + 1\right)$; $print\left(\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12}{\sqrt{Q_{shape}}} \cdot \left(\frac{d_{diameter}}{2 \cdot B_{thickness}} + 1\right)\right)$; $Q := \frac{\pi^2}{4} \cdot \left(\frac{3}{4} + \left(\frac{a}{2 \cdot c}\right)^2\right)^2 - \frac{7}{33} \cdot \left(\frac{\sigma}{\sigma_{ys}}\right)^2$; $\text{sigma} := \frac{P \cdot d}{2 \cdot B}$;

$print\left(\text{" (3) The hoop stress } \sigma \text{ "} = \frac{P_{pressure} \cdot d_{diameter}}{2 \cdot B_{thickness}}\right)$; ;

$print($" (4) The shape factor Q " $= Q_{shape})$; $printf($"\n%s", " Read the data from the temperature mechanical properties table"$)$; $printf($"\n%s", " "$)$;;

$print(TYPE)$;**for** i **from** 1 **to** 5 **do** $print(s[i] = A[k][i])$; $s[i] := A[k][i]$; **od**; ;;

$print\big(Material = s[1], \Sigma_{ys} = s[2], \Sigma_{ts} = s[3], KIc = s[4], E = s[5]\big)$; ;; $printf($"\n%s", " Calculating the hoop stress sigma , the shape factor Q , correction factor alpha and the total stress intensity factor KI "$)$; $printf($"\n%s", " "$)$; $sigma1 := evalf\left(\frac{P1 \cdot d1}{2 \cdot Bnew}, 5\right)$;

$print($" From (3) "$)$; $print\left(\text{" (3) The hoop stress } \sigma \text{ "} = \frac{P_{pressure} \cdot d_{diameter}}{2 \cdot B_{thickness}}\right)$;

$print($" σ " $= sigma1)$;; $printf($"\n%s", " "$)$; $print($" From (4) "$)$;

$print($" (4) The shape factor Q " $= Q_{shape})$; $\sigma_{ys} := s[2]$; $Q1 := \frac{\pi^2}{4} \cdot \left(\frac{3}{4} + \left(\frac{anew}{2 \cdot cnew}\right)^2\right)^2 - \frac{7}{33} \cdot \left(\frac{\sigma 1}{\sigma_{ys}}\right)^2$; $Q1 := evalf(Q1, 3)$; ; $print($" Q " $= Q1)$; ;

$printf($"\n%s", " "$)$; $print($" From (2) "$)$; $print\left(\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12}{\sqrt{Q_{shape}}} \cdot \left(\frac{d_{diameter}}{2 \cdot B_{thickness}} + 1\right)\right)$; $alpha1 := \frac{1.12}{\sqrt{Q1}} \cdot \left(\frac{d1}{2 \cdot Bnew} + 1\right)$; $alpha1 := evalf(alpha1, 3)$;

$print($" α " $= alpha1)$; ; $printf($"\n%s", " "$)$; $print($" From (1) "$)$;

$print\big($ " (1) The total stress intensity factor KI " $= \text{alpha} \cdot P \cdot \sqrt{\pi \cdot a}\big)$; $KI1 := alpha1 \cdot P1 \cdot \sqrt{\pi \cdot anew}$; $print(KI = KI1)$; $KI1 := evalf(KI1, 4)$; $print(KI = KI1)$; ; $printf($"\n%s", " "$)$; $printf($"\n%s", " 1/. Compare the total stress intensity factor KI and KIc "$)$; ;

$printf($"\n%s", " "$)$; $KIC := s[4]$; $print(KIc = KIC)$;

$print($" The fracture will not occur / the crack propagation is stable , as KI $<$ KIc " $)$;

$ineq := KI1 < KIC$; $print($" Analytical inequality ", $ineq)$; $KI2 := convert(taylor(KI1, t$

```
=t0), polynom); print(" Taylor polynomial expression of  KI   at  t   =  ", t0, KITaylor
=KI2); ineq2 := convert(KI2, polynom) < KIC;
print("  The analytical inequality obtained by using Taylor polynomial expression ",
ineq2); st2 := solve( {ineq2, t > 0, t < tlim}, t) ;
print(" Estimating time-limit  based on Taylor polynomial expression of  KI  ", st2);
printf("\n%s", "    "); st := solve( {ineq, t > 0, t < tlim}, t) ;
print(" Estimating time−limit  based on  analytical method  for  KI  ", st); printf("\n%s",
"    "); printf("\n%s",
" 2/. Determine the maximum pressure Pmax that causes the crack propagation "); ;
printf("\n%s", "    "); print(" From (1) "); print(  " (1)  The critical stress intensity  KIc "
=α[c]·P[Max]·sqrt(π·a)); P[MAX] := KIC/(α1·sqrt(π·anew)); P[MAX] := evalf(P[MAX], 5); print(Pmax
=P[MAX]); ; printf("\n%s", "    "); printf("\n%s",
" 3/. Consider the relation between Q and α  "); ; printf("\n%s", "    ");
print(" From (2) "); print( " Substitute  x  for  alpha   and   y for  Q  into  (2) ");
print(alpha=alphax); x=evalf((1.12/sqrt(y))·(d1/(2·Bnew) + 1), 4) ; print(%)   ;; printf("\n%s",
"    ");; printf("\n%s", "    ");
print(" Graphical relations between t  and stress intensity factor (plot1) _ Taylor series
expansion of stress intensity factor KI (plot2) " );; print(" End of the program " );;;;
printf("\n%s", "    "); plot( [KI1 , KI2], t=0 ..tlim, y=y0 ..y1, axes=boxed, labels=[ `t `,
`KI `], color=[blue, green], legend=[KIplot1, KITaylorplot2], thickness=[3, 3], linestyle
=[solid, dot]);                    end:
> pressure[KIandKIc] :=proc(tKcor, ymin, ymax)  local pK ;
print(" The value of  KIc is given from the engineering design requirement ( see table 1. ) :
KIc  = ", KIC);
print(" The expresion of continous or discrete form of KI and KITaylor  ");
print(" KI = ", KI(t) =KI1); print(" KITaylor = ", KITaylor(t) =KI2);
                    ; plot( [KI1, KI2, KIC], t=0 .. tKcor, y=ymin ..ymax, color=[blue,
green, red], thickness=[3, 3, 3], legend=[ `blueKI1`, `greenKITaylor`, `redKIc`], linestyle
=[solid, dot, dash]);      end:
> pressure[Plotinput] :=proc(tcor, ymin, ymax)   local pp ;
print(" The expresion of continous or discrete form of internal pressure ");
print(" Pressure function = ", f(t) =P2);                       ; plot(P2, t=0 .. tcor, y
=ymin ..ymax, color=brown, thickness=3, legend= `brownP`);      end :
```

Example 3.2.1

Consider a round vessel with technical data below :
one-half crack depth a = 5mm ; thickness of vessel B = 25cm ; one-half crack length c = 8mm ; internal diameter d = 0.8m ; internal pressure P = $16.13084112 \ln(t + 15.22) + 2.5\, e^{\sin(0.06507451031\, t + 0.6642806012) - 9600.268}$ MPa ; type : Ti6Al4V2 i.e A[4][1]

a. Find the expression of stress intensity factor KI and Taylor polynomial of KI .
b. Evaluating the time-limit for applying the continous / discrete form of internal dynamic pressure .

Solution :
Run the procedure $pressure_{Estm}(, , ,);$

$> pressure_{Estm}(5, 25, 8, 0.8, 16.13084112 \ln(t + 15.22) + 2.5 e^{\sin(0.06507451031 t + 0.6642806012) - 9600.268}, A[4][1], 4, 10000, 5000, 0, 80);$

" Start the program "

" Parameters of the metal vessel and internal pressure P : ", $a = 5, B = 25, 2c = 16, d = 0.8, P = 16.13084112 \ln(t + 15.22) + 2.5 e^{\sin(0.06507451031 t + 0.6642806012) - 9600.268}, type = Ti6Al4V2, \frac{1}{2}\frac{a}{c} = 0.312, row = 4$

" Convert the units into meter : "

$$a = \frac{1}{200}, B = \frac{1}{4}, c = \frac{1}{125}, d = 0.8$$

$$\text{" (1) The total stress intensity factor KI "} = \alpha P_{pressure} \sqrt{\pi a}$$

$$\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12 \left(\frac{1}{2} \frac{d_{diameter}}{B_{thickness}} + 1\right)}{\sqrt{Q_{shape}}}$$

$$\text{" (3) The hoop stress } \sigma \text{ "} = \frac{1}{2} \frac{P_{pressure} d_{diameter}}{B_{thickness}}$$

$$\text{" (4) The shape factor Q "} = \left(\frac{1}{4} \pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2}\right)^2 - \frac{7}{33} \frac{\sigma^2}{\sigma_{ys}^2}\right)_{shape}$$

```
Read the data from the temperature mechanical properties table
```

$TYPE$

$s_1 = Ti6Al4V2$

$s_2 = 1007$

$s_3 = 1034$

$s_4 = 40$

$s_5 = 130$

$$Material = Ti6Al4V2, \Sigma_{ys} = 1007, \Sigma_{ts} = 1034, KIc = 40, E = 130$$

```
Calculating the hoop stress  sigma , the shape factor Q ,
correction factor alpha  and the total stress intensity factor
KI
```

" From (3) "

$$\text{" (3) The hoop stress } \sigma \text{ "} = \frac{1}{2}\,\frac{P_{pressure}\,d_{diameter}}{B_{thickness}}$$

$$\text{" } \sigma \text{ "} = 25.810\ln(t+15.22) + 4.0000\,\mathrm{e}^{\sin(0.065075\,t+0.66428)} - 9600.3$$

" From (4) "

$$\text{" (4) The shape factor Q "} = \left(\frac{1}{4}\,\pi^2\left(\frac{3}{4}+\frac{1}{4}\,\frac{a^2}{c^2}\right)^2 - \frac{7}{33}\,\frac{\sigma^2}{\sigma_{ys}^2}\right)_{shape}$$

$$\text{" Q "} = 1.77 - 2.09\;10^{-7}\left(25.8\ln(t+15.2) + 4.00\,\mathrm{e}^{\sin(0.0651\,t+0.664)} - 9600.\right)^2$$

" From (2) "

$$\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12\left(\frac{1}{2}\,\frac{d_{diameter}}{B_{thickness}}+1\right)}{\sqrt{\left(\frac{1}{4}\,\pi^2\left(\frac{3}{4}+\frac{1}{4}\,\frac{a^2}{c^2}\right)^2 - \frac{7}{33}\,\frac{\sigma^2}{\sigma_{ys}^2}\right)_{shape}}}$$

$$\text{" } \alpha \text{ "} = \frac{2.91}{\sqrt{1.77 - 2.09\;10^{-7}\left(25.8\ln(t+15.2) + 4.00\,\mathrm{e}^{\sin(0.0651\,t+0.664)} - 9600.\right)^2}}$$

" From (1) "

$$\text{" (1) The total stress intensity factor KI "} = \alpha\,P\,\sqrt{\pi\,a}$$

$$KI = \left(0.1455000000\left(16.13084112\ln(t+15.22) + 2.5\,\mathrm{e}^{\sin(0.06507451031\,t+0.6642806012)} - 9600.268\right)\sqrt{2}\,\sqrt{\pi}\right) \Big/ \sqrt{1.77 - 2.09\;10^{-7}\left(25.8\ln(t+15.2) + 4.00\,\mathrm{e}^{\sin(0.0651\,t+0.664)} - 9600.\right)^2}$$

$$KI = \frac{0.3648\left(16.13\ln(t+15.22) + 2.5\,\mathrm{e}^{\sin(0.06507\,t+0.6643)} - 9600.\right)}{\sqrt{1.77 - 2.09\;10^{-7}\left(25.8\ln(t+15.2) + 4.00\,\mathrm{e}^{\sin(0.0651\,t+0.664)} - 9600.\right)^2}}$$

```
1/. Compare the  total stress intensity factor KI  and KIc
```

$$KIc = 40$$

" The fracture will not occur / the crack propagation is stable , as KI $<$ KIc "

" Analytical inequality ",

$$\frac{0.3648\left(16.13\ln(t+15.22)+2.5\,e^{\sin(0.06507\,t+0.6643)-9600.}\right)}{\sqrt{1.77-2.09\,10^{-7}\left(25.8\ln(t+15.2)+4.00\,e^{\sin(0.0651\,t+0.664)-9600.}\right)^2}} < 40$$

" Taylor polynomial expression of KI at t = ", 5000, $KITaylor = 33.34426980 + 0.0008894882453\,t - 8.849970677\,10^{-8}\,(t-5000)^2 + 1.175367970\,10^{-11}\,(t-5000)^3 - 1.756728539\,10^{-15}\,(t-5000)^4 + 2.801127987\,10^{-19}\,(t-5000)^5$

" The analytical inequality obtained by using Taylor polynomial expression ", $0.0008894882453\,t - 8.849970677\,10^{-8}\,(t-5000)^2 + 1.175367970\,10^{-11}\,(t-5000)^3 - 1.756728539\,10^{-15}\,(t-5000)^4 + 2.801127987\,10^{-19}\,(t-5000)^5 < 6.65573020$

" Estimating time-limit based on Taylor polynomial expression of KI ", $\{0. < t, t < 8155.424162\}$

" Estimating time-limit based on analytical method for KI "

```
2/. Determine the maximum pressure Pmax that causes the crack
propagation
```

" From (1) "

" (1) The critical stress intensity KIc " $= \alpha_c\, P_{Max} \sqrt{\pi\, a}$

$$Pmax = 109.67\sqrt{1.77-2.09\,10^{-7}\left(25.8\ln(t+15.2)+4.00\,e^{\sin(0.0651\,t+0.664)-9600.}\right)^2}$$

```
3/. Consider the relation between Q and &alpha;
```

" From (2) "

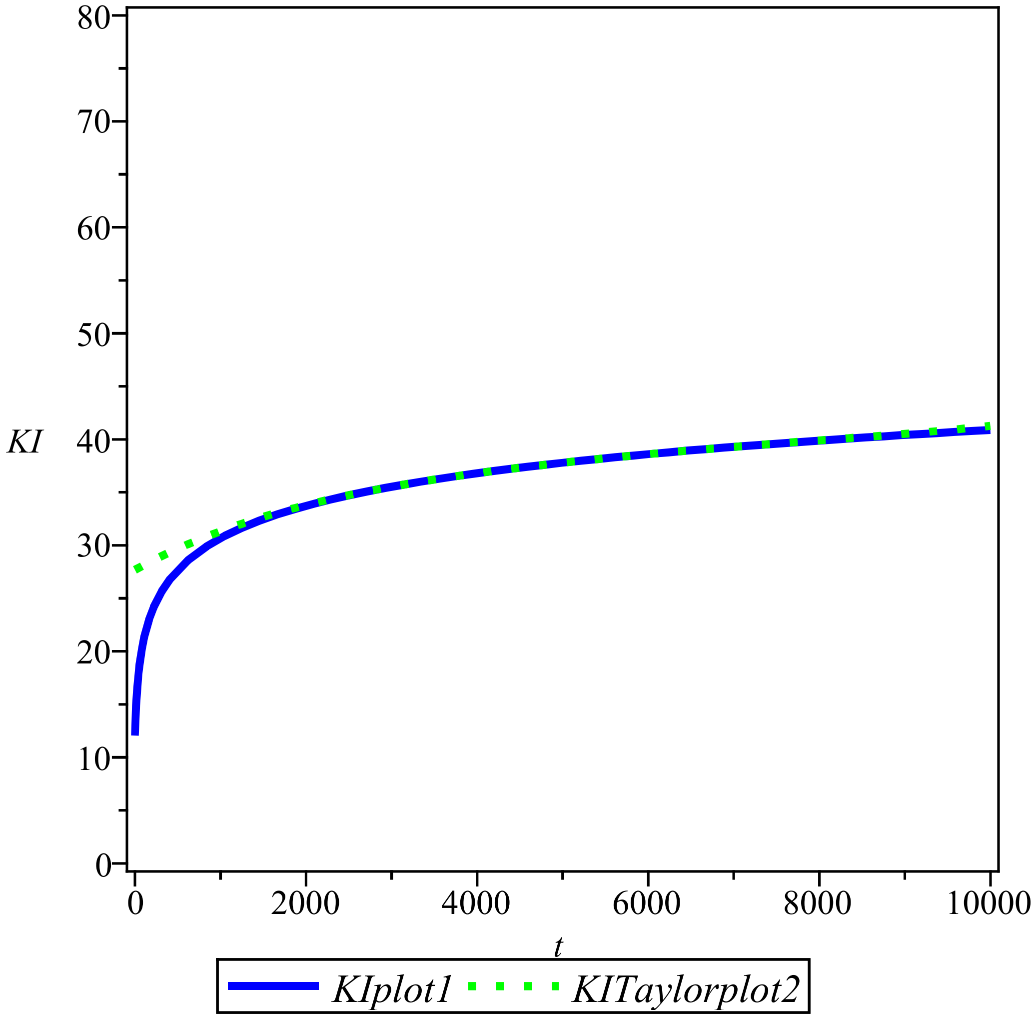
" Substitute x for alpha and y for Q into (2) "

$$\alpha = \frac{1.12\left(\frac{1}{2}\,\frac{d_{diameter}}{B_{thickness}}+1\right)}{\sqrt{Q_{shape}}}$$

$$x = \frac{2.912}{\sqrt{y}}$$

" Graphical relations between t and stress intensity factor (plot1) _ Taylor series expansion of stress intensity factor KI (plot2) "

" End of the program "



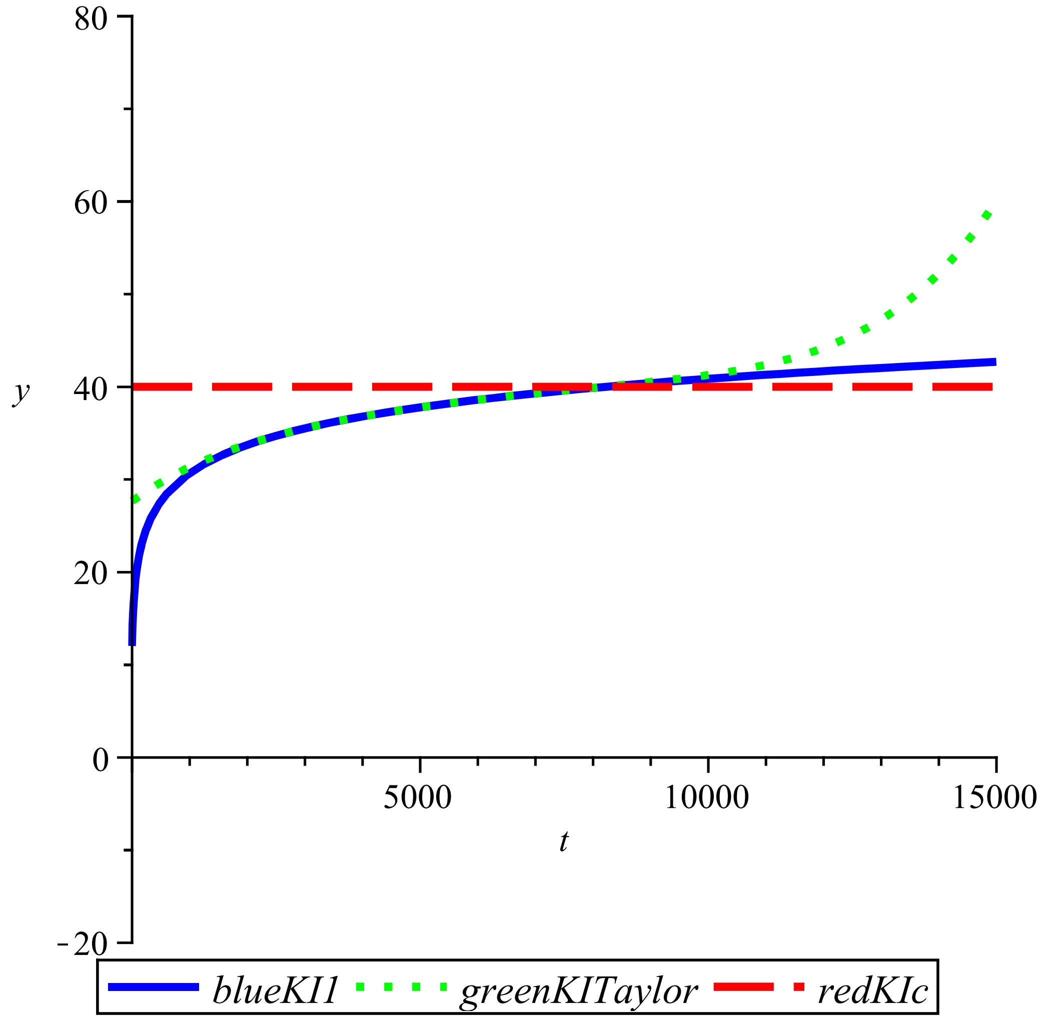
> $pressure_{KIandKIc}(15000, -20, 80)$

" The value of KIc is given from the engineering design requirement (see table 1.) : KIc = ", 40

" The expresion of continous or discrete form of KI and KITaylor "

$$\text{" KI = ", } KI(t) = \frac{0.3648\,\left(16.13\,\ln(t+15.22) + 2.5\,\mathrm{e}^{\sin(0.06507\,t+0.6643)\,-\,9600.}\right)}{\sqrt{1.77 - 2.09\,10^{-7}\,\left(25.8\,\ln(t+15.2) + 4.00\,\mathrm{e}^{\sin(0.0651\,t+0.664)\,-\,9600.}\right)^{2}}}$$

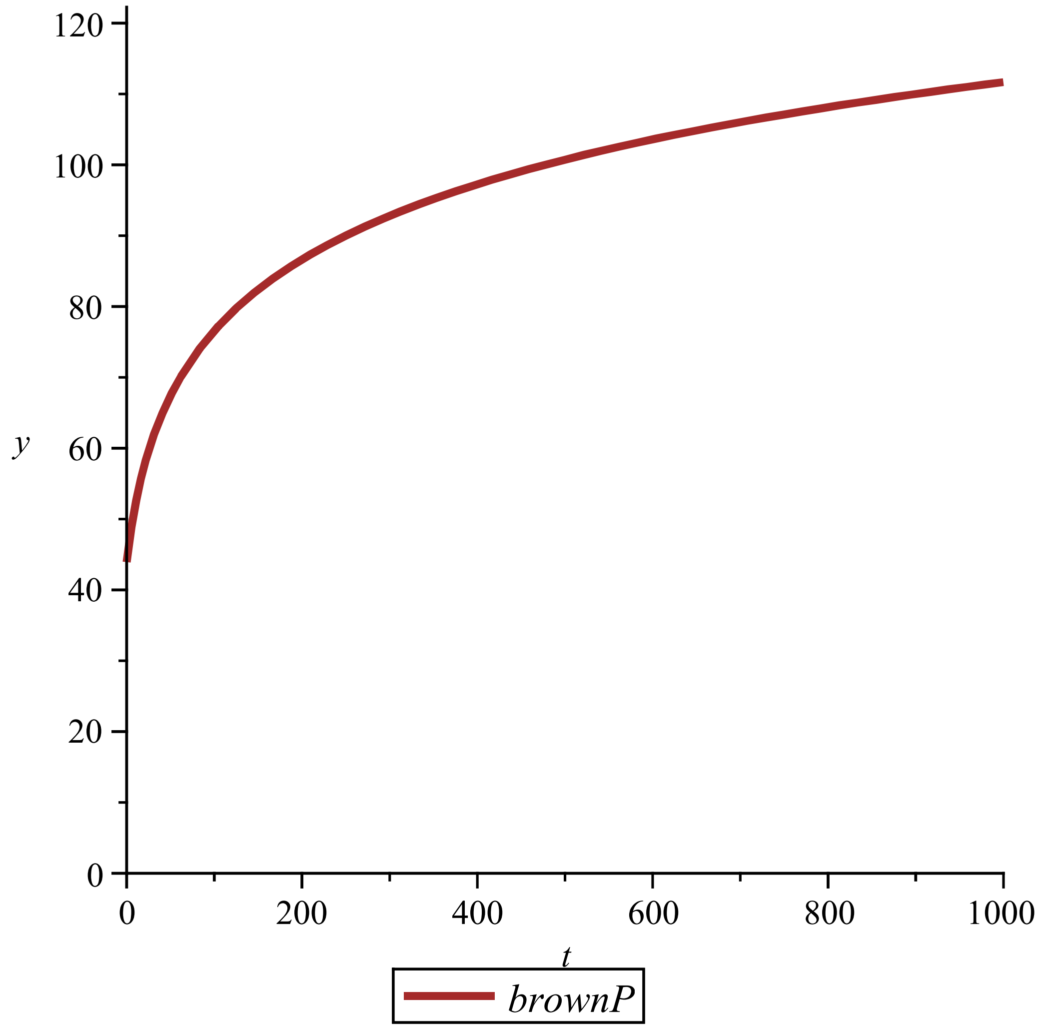
$$\text{" KITaylor = ", } KITaylor(t) = 33.34426980 + 0.0008894882453\,t - 8.849970677\,10^{-8}\,(t-5000)^{2} + 1.175367970\,10^{-11}\,(t-5000)^{3} - 1.756728539\,10^{-15}\,(t-5000)^{4} + 2.801127987\,10^{-19}\,(t-5000)^{5}$$



> $pressure_{Plotinput}(1000, 00, 122.28)$;

" The expresion of continous or discrete form of internal pressure "

" Pressure function = ", $f(t) = 16.13084112 \ln(t + 15.22) + 2.5\, e^{\sin(0.06507451031\, t + 0.6642806012)} - 9600.268$



Example 3.2.2

Consider a round vessel with technical data below :
one-half crack depth a = 5mm ; thickness of vessel B = 25cm ; one-half crack length c = 8mm ; internal diameter d = 0.8m ; internal pressure P is given from the data table below ; type : 18Ni (250) i.e A[9][1]

18Ni (250)	*adjust coeff*	1/ 948\ .89\ 2									
t	0	5	10	15	20	25	30	35	40	45	50
MPa	0	150	170	140	165	220	340	800	920	900	820

a. Find the expression of stress intensity factor KI and Taylor polynomial of KI .
b. Evaluating the time-limit for applying the discrete form of internal dynamic pressure P .

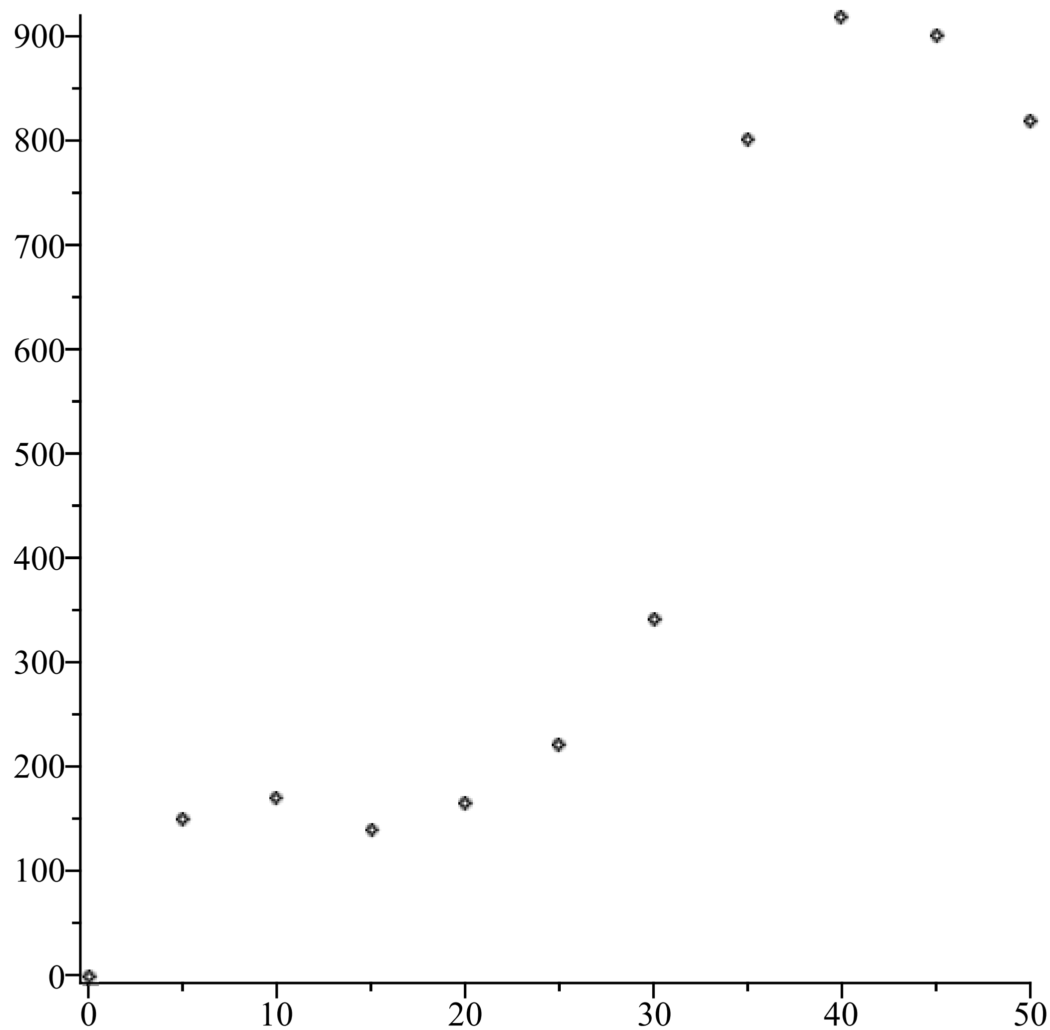
Solution :

Fitting the data points by
$fnumPower := PolynomialInterpolation([[0, 0.0], [5, 150], [10, 170], [15, 140], [20, 165], [25, 220], [30, 340], [35, 800], [40, 920], [45\ , 900\ ], [50\ , 820\ ]], t, form = power);$

> $with(CurveFitting) : with(plots) : fnumPower := PolynomialInterpolation([[0, 0.0], [5, 150], [10, 170], [15, 140], [20, 165], [25, 220], [30, 340], [35, 800], [40, 920], [45\ , 900\ ], [50\ , 820\ ]], t, form = power);$

$$fnumPower := -6.439506133\ 10^{-10}\ t^{10} + 1.544550255\ 10^{-7}\ t^9 - 0.00001579195757\ t^8 + 0.0008993206290\ t^7 - 0.03128031829\ t^6 + 0.6852472173\ t^5 - 9.393835424\ t^4 + 77.20204438\ t^3 - 344.1569100\ t^2 + 651.5571380\ t \qquad \textbf{(3)}$$

> $pointplot(\{[0, 0.0], [5, 150], [10, 170], [15, 140], [20, 165], [25, 220], [30, 340], [35, 800], [40, 920], [45\ , 900\ ], [50\ , 820\ ]\}, axes = framed);$



Run this procedure $pressure_{Estm}(, , ,)$ with the approximate polynomial *fnumPower*

> $pressure_{Estm}\left(5, 25, 8, 0.8, \frac{fnumPower}{948.892}, A[9][1], 9, 80, 47, -50, 100\right);$

$$\text{" Start the program "}$$

" Parameters of the metal vessel and internal pressure P : ", $a = 5$, $B = 25$, $2\,c = 16$, $d = 0.8$, $P = -6.786342525\;10^{-13}\,t^{10} + 1.627740833\;10^{-10}\,t^{9} - 1.664252366\;10^{-8}\,t^{8} + 9.477586793\;10^{-7}\,t^{7} - 0.00003296509854\,t^{6} + 0.0007221551210\,t^{5} - 0.009899794098\,t^{4} + 0.08136020154\,t^{3} - 0.3626934466\,t^{2} + 0.6866504700\,t$, $type = 18\,Ni250$, $\frac{1}{2}\,\frac{a}{c} = 0.312$, $row = 9$

$$\text{" Convert the units into meter : "}$$

$$a = \frac{1}{200},\ B = \frac{1}{4},\ c = \frac{1}{125},\ d = 0.8$$

$$\text{" (1) The total stress intensity factor KI "} = \alpha\, P_{pressure}\sqrt{\pi\, a}$$

$$\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12\left(\frac{1}{2}\,\frac{d_{diameter}}{B_{thickness}} + 1\right)}{\sqrt{Q_{shape}}}$$

$$\text{" (3) The hoop stress } \sigma \text{ "} = \frac{1}{2}\,\frac{P_{pressure}\,d_{diameter}}{B_{thickness}}$$

$$\text{" (4) The shape factor Q "} = \left(\frac{1}{4}\,\pi^2\left(\frac{3}{4} + \frac{1}{4}\,\frac{a^2}{c^2}\right)^2 - \frac{7}{33}\,\frac{\sigma^2}{\sigma_{ys}^2}\right)_{shape}$$

```
Read the data from the temperature mechanical properties table
```

$$TYPE$$

$$s_1 = 18\,Ni250$$

$$s_2 = 1290$$

$$s_3 = 1345$$

$$s_4 = 176$$

$$s_5 = NA$$

$$Material = 18\,Ni250,\ \Sigma_{ys} = 1290,\ \Sigma_{ts} = 1345,\ KIc = 176,\ E = NA$$

```
 Calculating the hoop stress  sigma , the shape factor Q ,
correction factor alpha  and the total stress intensity factor
KI
```

$$\text{" From (3) "}$$

$$\text{" (3) The hoop stress } \sigma \text{ "} = \frac{1}{2}\,\frac{P_{pressure}\,d_{diameter}}{B_{thickness}}$$

$\text{" } \sigma \text{ "} = -1.0858\;10^{-12}\,t^{10} + 2.6043\;10^{-10}\,t^{9} - 2.6629\;10^{-8}\,t^{8} + 0.0000015164\,t^{7}$

$$- 0.000052745\, t^6 + 0.0011554\, t^5 - 0.015840\, t^4 + 0.13018\, t^3 - 0.58030\, t^2 + 1.0986\, t$$

" From (4) "

$$\text{" (4) The shape factor Q "} = \left(\frac{1}{4} \pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2} \right)^2 - \frac{7}{33} \frac{\sigma^2}{\sigma_{ys}^2} \right)_{shape}$$

$$\text{" Q "} = 1.77 - 1.27\, 10^{-7} \left(-1.09\, 10^{-12}\, t^{10} + 2.60\, 10^{-10}\, t^9 - 2.66\, 10^{-8}\, t^8 + 0.00000152\, t^7 - 0.0000527\, t^6 + 0.00116\, t^5 - 0.0158\, t^4 + 0.130\, t^3 - 0.580\, t^2 + 1.10\, t \right)^2$$

" From (2) "

$$\text{"(2) Correction factor } \alpha \text{ "} = \frac{1.12 \left(\frac{1}{2} \frac{d_{diameter}}{B_{thickness}} + 1 \right)}{\sqrt{\left(\frac{1}{4} \pi^2 \left(\frac{3}{4} + \frac{1}{4} \frac{a^2}{c^2} \right)^2 - \frac{7}{33} \frac{\sigma^2}{\sigma_{ys}^2} \right)_{shape}}}$$

$$\text{" } \alpha \text{ "} = 2.91 \Big/ \left(1.77 - 1.27\, 10^{-7} \left(-1.09\, 10^{-12}\, t^{10} + 2.60\, 10^{-10}\, t^9 - 2.66\, 10^{-8}\, t^8 + 0.00000152\, t^7 - 0.0000527\, t^6 + 0.00116\, t^5 - 0.0158\, t^4 + 0.130\, t^3 - 0.580\, t^2 + 1.10\, t \right)^2 \right)^{1/2}$$

" From (1) "

$$\text{" (1) The total stress intensity factor KI "} = \alpha\, P \sqrt{\pi\, a}$$

$$KI = \Big(0.1455000000 \left(-6.786342525\, 10^{-13}\, t^{10} + 1.627740833\, 10^{-10}\, t^9 - 1.664252366\, 10^{-8}\, t^8 + 9.477586793\, 10^{-7}\, t^7 - 0.00003296509854\, t^6 + 0.0007221551210\, t^5 - 0.009899794098\, t^4 + 0.08136020154\, t^3 - 0.3626934466\, t^2 + 0.6866504700\, t \right) \sqrt{2} \sqrt{\pi} \Big) \Big/ \left(1.77 - 1.27\, 10^{-7} \left(-1.09\, 10^{-12}\, t^{10} + 2.60\, 10^{-10}\, t^9 - 2.66\, 10^{-8}\, t^8 + 0.00000152\, t^7 - 0.0000527\, t^6 + 0.00116\, t^5 - 0.0158\, t^4 + 0.130\, t^3 - 0.580\, t^2 + 1.10\, t \right)^2 \right)^{1/2}$$

$$KI = \Big(0.3648 \left(-6.786\, 10^{-13}\, t^{10} + 1.628\, 10^{-10}\, t^9 - 1.664\, 10^{-8}\, t^8 + 9.478\, 10^{-7}\, t^7 - 0.00003297\, t^6 + 0.0007222\, t^5 - 0.009900\, t^4 + 0.08136\, t^3 - 0.3627\, t^2 + 0.6867\, t \right) \Big) \Big/$$

$$\left(1.77 - 1.27\ 10^{-7} \left(-1.09\ 10^{-12} t^{10} + 2.60\ 10^{-10} t^{9} - 2.66\ 10^{-8} t^{8} + 0.00000152\ t^{7} - 0.0000527\ t^{6} + 0.00116\ t^{5} - 0.0158\ t^{4} + 0.130\ t^{3} - 0.580\ t^{2} + 1.10\ t\right)^{2}\right)^{1/2}$$

```
1/. Compare the  total stress intensity factor KI  and KIc
```

$$KIc = 176$$

" The fracture will not occur / the crack propagation is stable , as KI $<$ KIc "

" Analytical inequality ", $\left(0.3648 \left(-6.786\ 10^{-13} t^{10} + 1.628\ 10^{-10} t^{9} - 1.664\ 10^{-8} t^{8} + 9.478\ 10^{-7} t^{7} - 0.00003297\ t^{6} + 0.0007222\ t^{5} - 0.009900\ t^{4} + 0.08136\ t^{3} - 0.3627\ t^{2} + 0.6867\ t\right)\right) \Big/ \left(1.77 - 1.27\ 10^{-7} \left(-1.09\ 10^{-12} t^{10} + 2.60\ 10^{-10} t^{9} - 2.66\ 10^{-8} t^{8} + 0.00000152\ t^{7} - 0.0000527\ t^{6} + 0.00116\ t^{5} - 0.0158\ t^{4} + 0.130\ t^{3} - 0.580\ t^{2} + 1.10\ t\right)^{2}\right)^{1/2} < 176$

" Taylor polynomial expression of KI at t = ", 47, $KITaylor = -2377.576003 + 51.69382641\ t + 63.00700353\ (t - 47)^{2} + 88.08829310\ (t - 47)^{3} + 130.3049045\ (t - 47)^{4} + 198.6925598\ (t - 47)^{5}$

" The analytical inequality obtained by using Taylor polynomial expression ", $51.69382641\ t + 63.00700353\ (t - 47)^{2} + 88.08829310\ (t - 47)^{3} + 130.3049045\ (t - 47)^{4} + 198.6925598\ (t - 47)^{5} < 2553.576003$

" Estimating time-limit based on Taylor polynomial expression of KI ", $\{0. < t, t < 47.63775925\}$

" Estimating time-limit based on analytical method for KI ", $\{0. < t, t < 47.52889066\}$, $\{71.07919106 < t, t < 72.30034108\}$

```
2/. Determine the maximum pressure Pmax that causes the crack
propagation
```

" From (1) "

" (1) The critical stress intensity KIc " $= \alpha_c P_{Max} \sqrt{\pi a}$

$Pmax = 482.55 \left(1.77 - 1.27\ 10^{-7} \left(-1.09\ 10^{-12} t^{10} + 2.60\ 10^{-10} t^{9} - 2.66\ 10^{-8} t^{8}\right.\right.$

$$+ 0.00000152\, t^7 - 0.0000527\, t^6 + 0.00116\, t^5 - 0.0158\, t^4 + 0.130\, t^3 - 0.580\, t^2 + 1.10\, t)^2)$$

1/2

```
3/. Consider the relation between Q and &alpha;
```

" From (2) "

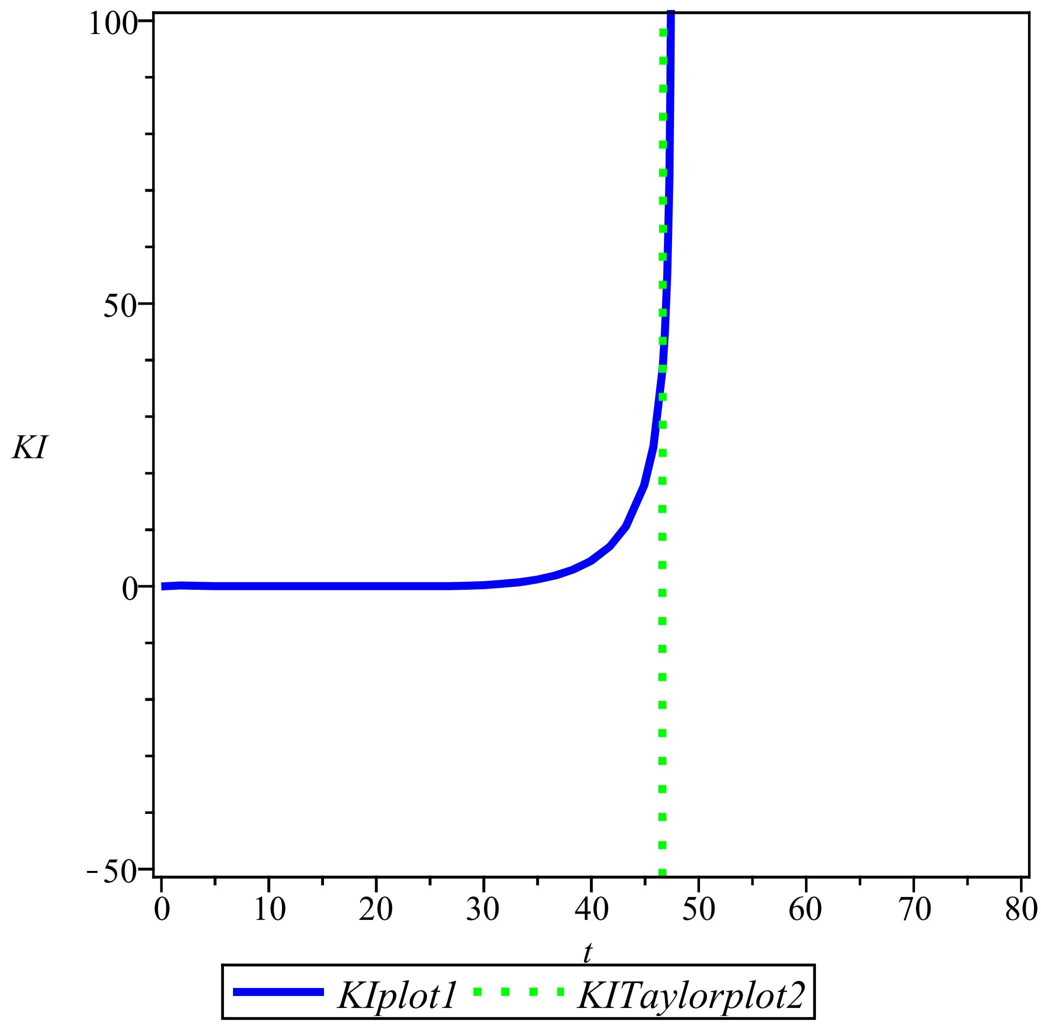
" Substitute x for alpha and y for Q into (2) "

$$\alpha = \frac{1.12 \left(\frac{1}{2} \frac{d_{diameter}}{B_{thickness}} + 1 \right)}{\sqrt{Q_{shape}}}$$

$$x = \frac{2.912}{\sqrt{y}}$$

" Graphical relations between t and stress intensity factor (plot1) _ Taylor series expansion of stress intensity factor KI (plot2) "

" End of the program "



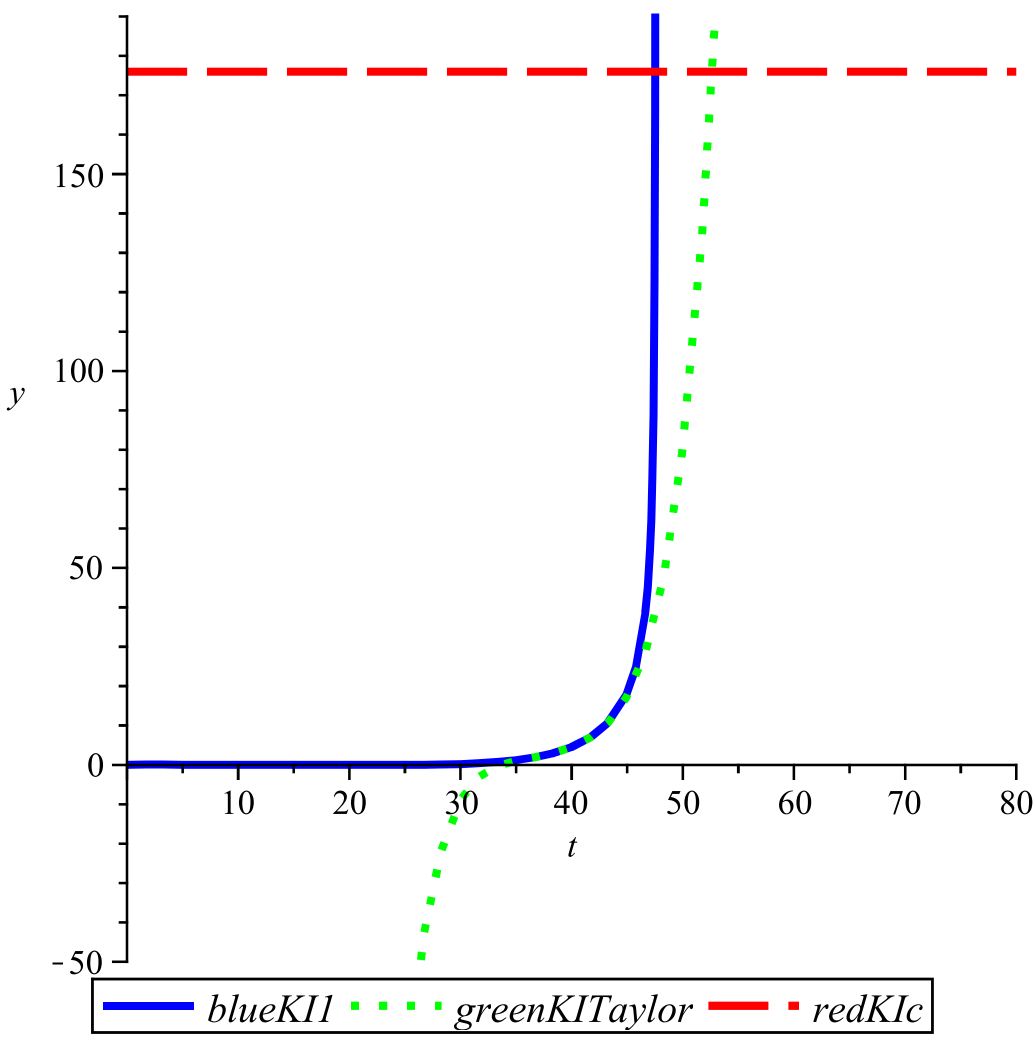
> $pressure_{KIandKIc}(80, -50, 190)$

" The value of KIc is given from the engineering design requirement (see table 1.) : KIc = ", 176

" The expresion of continous or discrete form of KI and KITaylor "

" KI = ", $KI(t) = \left(0.3648\left(-6.786\ 10^{-13}\ t^{10} + 1.628\ 10^{-10}\ t^9 - 1.664\ 10^{-8}\ t^8 + 9.478\ 10^{-7}\ t^7 - 0.00003297\ t^6 + 0.0007222\ t^5 - 0.009900\ t^4 + 0.08136\ t^3 - 0.3627\ t^2 + 0.6867\ t\right)\right) \Big/ \left(1.77 - 1.27\ 10^{-7}\left(-1.09\ 10^{-12}\ t^{10} + 2.60\ 10^{-10}\ t^9 - 2.66\ 10^{-8}\ t^8 + 0.00000152\ t^7 - 0.0000527\ t^6 + 0.00116\ t^5 - 0.0158\ t^4 + 0.130\ t^3 - 0.580\ t^2 + 1.10\ t\right)^2\right)^{1/2}$

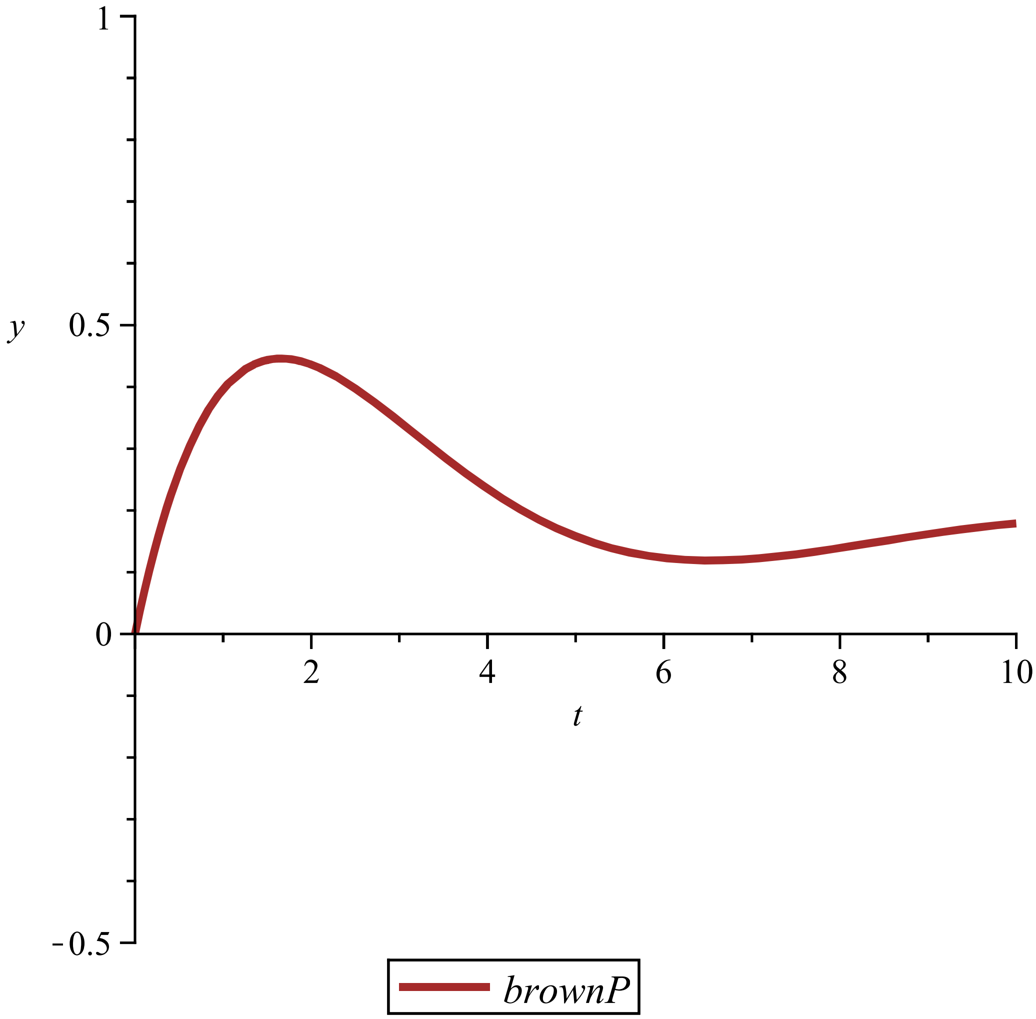
" KITaylor = ", $KITaylor(t) = -41.31589512 + 1.146168336\ t + 0.1478373550\ (t-40)^2 + 0.01574740070\ (t-40)^3 + 0.001656396686\ (t-40)^4 + 0.0001763158242\ (t-40)^5$



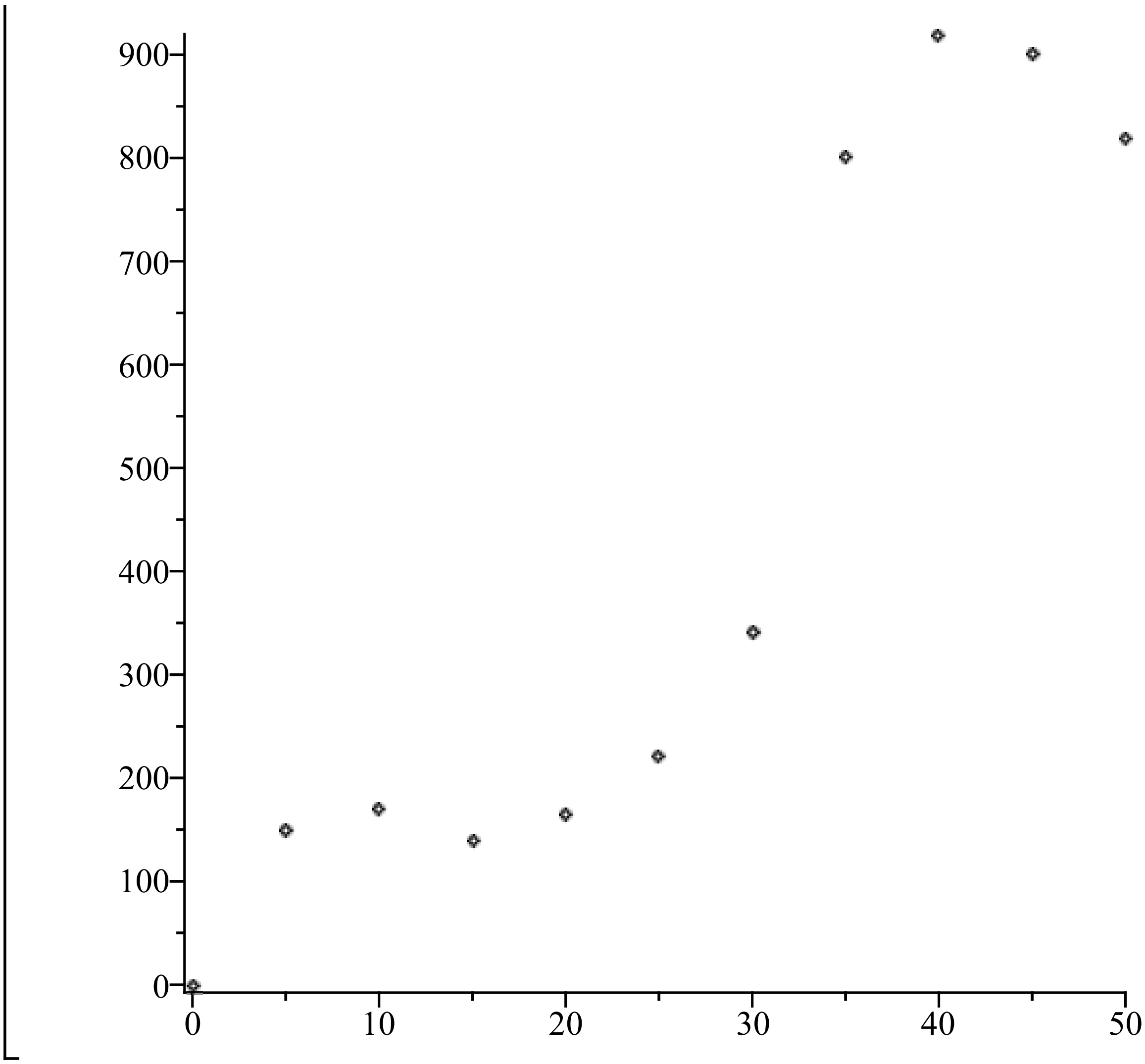
> $pressure_{Plotinput}(10, -0.5, 1)$; $pointplot(\{[0, 0.0], [5, 150], [10, 170], [15, 140], [20, 165], [25, 220], [30, 340], [35, 800], [40, 920], [45\ \ , 900\ \ ], [50\ \ , 820\ \ ]\}, axes = framed)$;

" The expresion of continous or discrete form of internal pressure "

" Pressure function = ", $f(t) = -6.786342525\ 10^{-13}\ t^{10} + 1.627740833\ 10^{-10}\ t^{9} - 1.664252366\ 10^{-8}\ t^{8} + 9.477586793\ 10^{-7}\ t^{7} - 0.00003296509854\ t^{6} + 0.0007221551210\ t^{5} - 0.009899794098\ t^{4} + 0.08136020154\ t^{3} - 0.3626934466\ t^{2} + 0.6866504700\ t$



1
y
0.5
0
-0.5
2
4
6
8
10
t
brownP

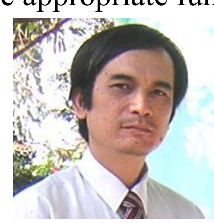


Notice that if we use Taylor series expansion of the approximate function for internal dynamic pressure at t = 47 it gives the same results of the time-limit calculated by analytical and series methods . ($0. < t, t < 47.63775925$, $0. < t, t < 47.52889066$)

Conclusion

Through the examples presented above, we find that the Maple tools has a lot of utilities in engineering computation and estimating the technical specifications . Note that in the case of applying internal dynamics pressure of discrete data we can use some softwares to find the best fitted functions , then get the expressions of the approximate functions to enter the procedures .

For the example 3.2.2 above, using the software Curve Expert , after entering the data we find the approximate expressions with parameters such as Standard Error, Correlation Coefficient ... From which we can choose the appropriate functions for the next calculation .



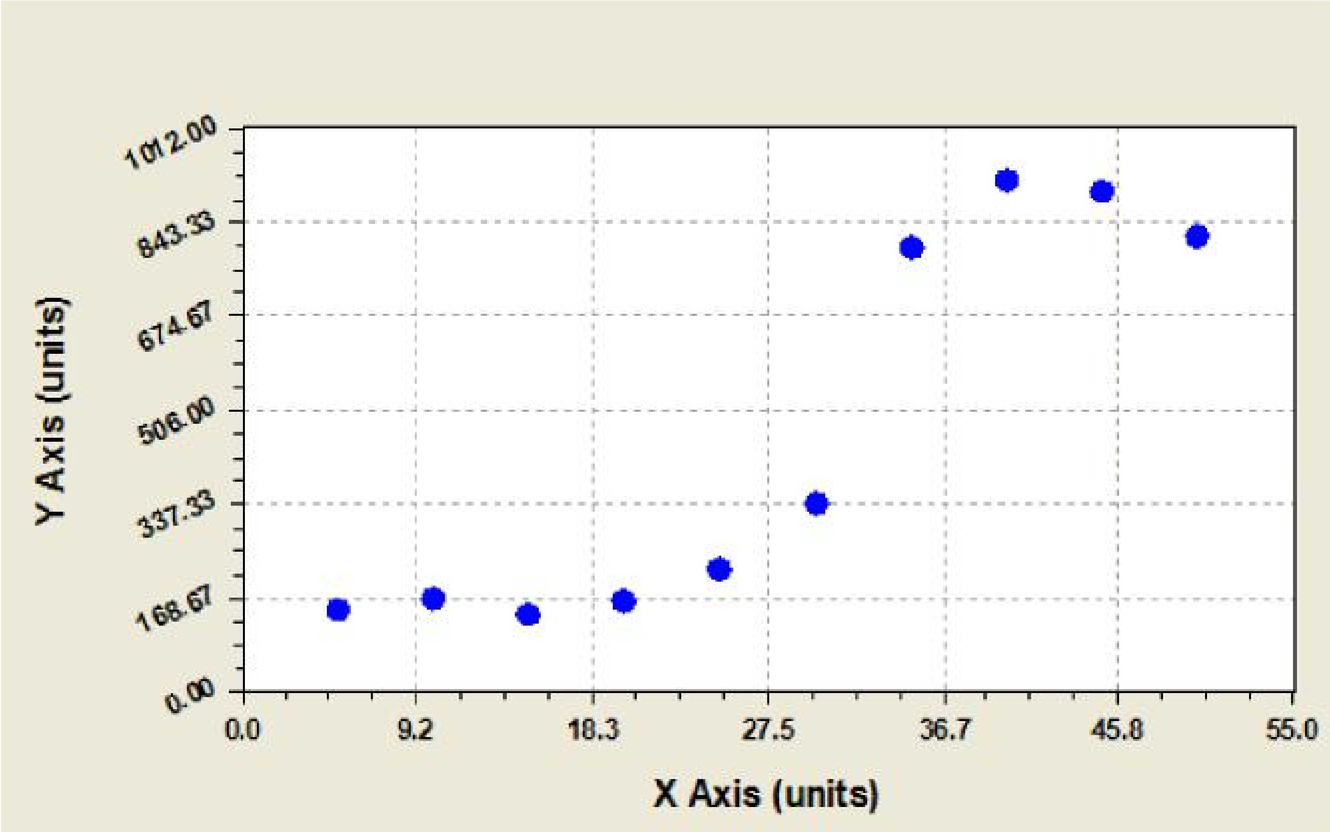


Fig 1. *Data plot* .

1. Logistic Model: y=a/(1+b*exp(-cx))
Standard Error: 114.9209641
Correlation Coefficient: 0.9580550
Comments:
The fit converged to a tolerance of 1e-006 in 36 iterations. No weighting used.

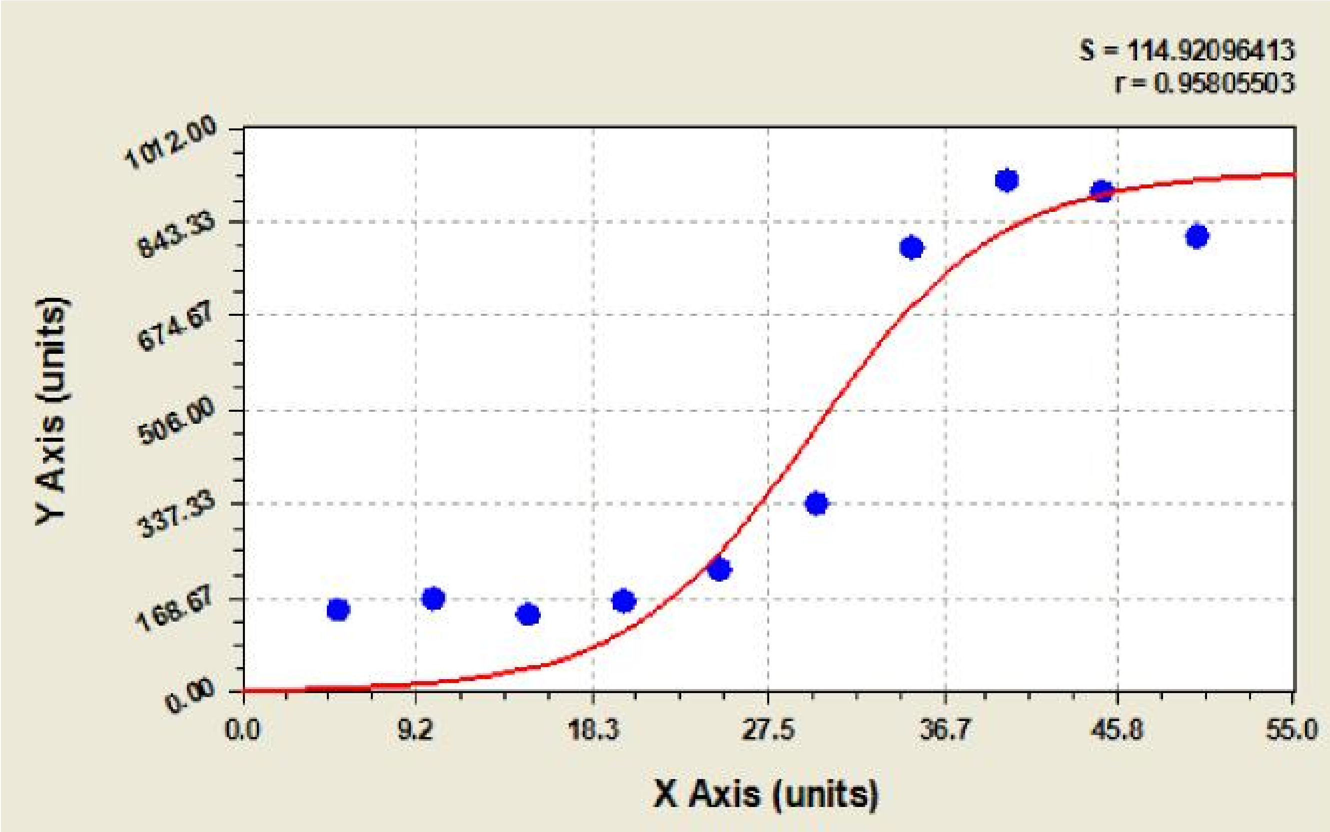


Fig 2 . *Graph of the logistic fit .*

2. The 4th Degree Polynomial Fit: y=a+bx+cx^2+dx^3+ex^4
Coefficient Data:

a =	-0.59440559
b =	57.013598
c =	-6.1678322
d =	0.23554002
e =	-0.0025734266

Standard Error: 75.4632160
Correlation Coefficient: 0.9866303
Comments:
Linear regression completed successfully. No weighting used.

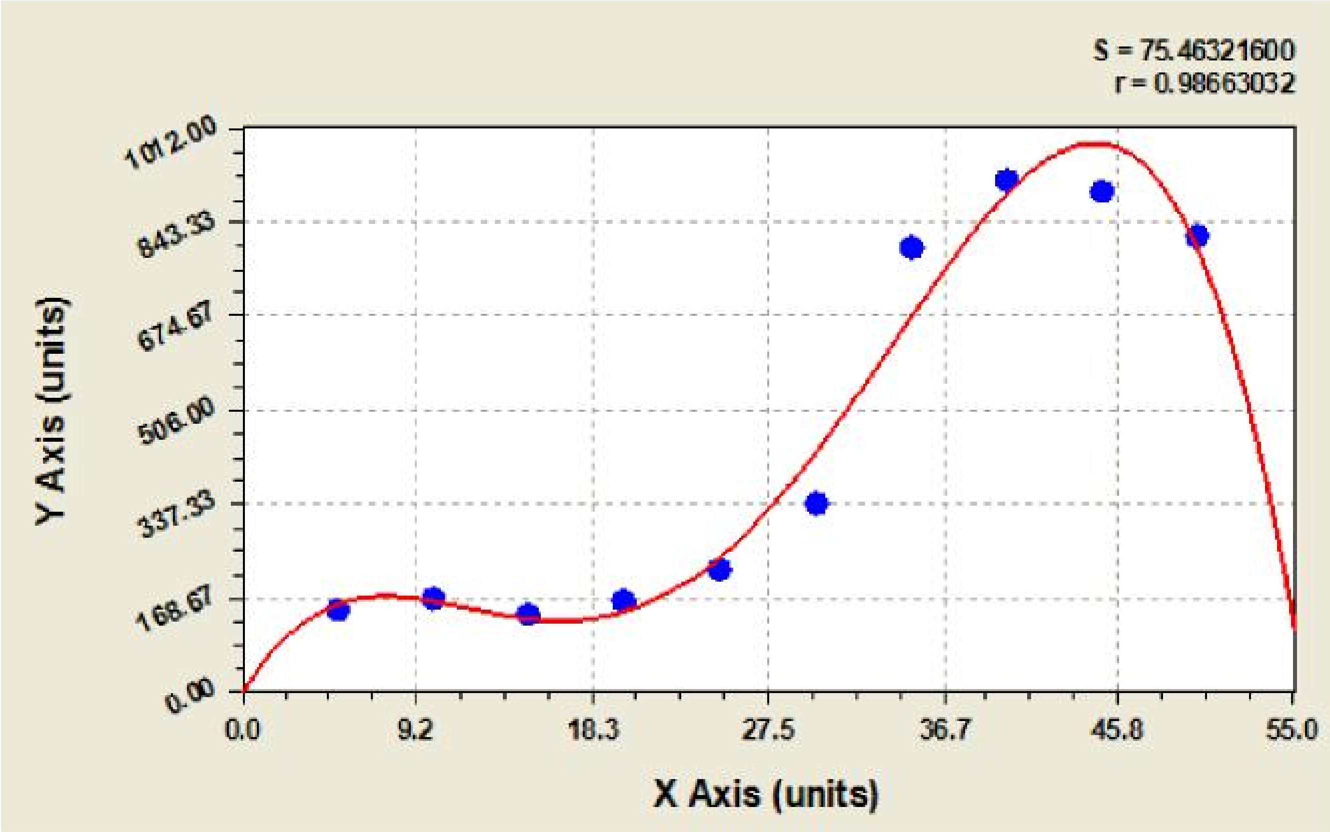


Fig 3 . *Graph of the 4th Degree Polynomial Fit* .

3. Lagrangian Interpolation: y=a+bx+cx^2+dx^3+ex^4+fx^5+gx^6+hx^7+ix^8+jx^9+kx^10
Coefficient Data:

a =	0
b =	651.55714
c =	-344.15691
d =	77.202045
e =	-9.3938355
f =	0.68524722
g =	-0.031280319
h =	0.00089932063
i =	-1.5791958e-005
j =	1.5445503e-007
k =	-6.4395062e-010

Standard Error: 0.0000000
Correlation Coefficient: 1.0000000
Comments:
Interpolation completed successfully. Since interpolations pass through each data point, the error is 0 and the corr. coeff. is 1.

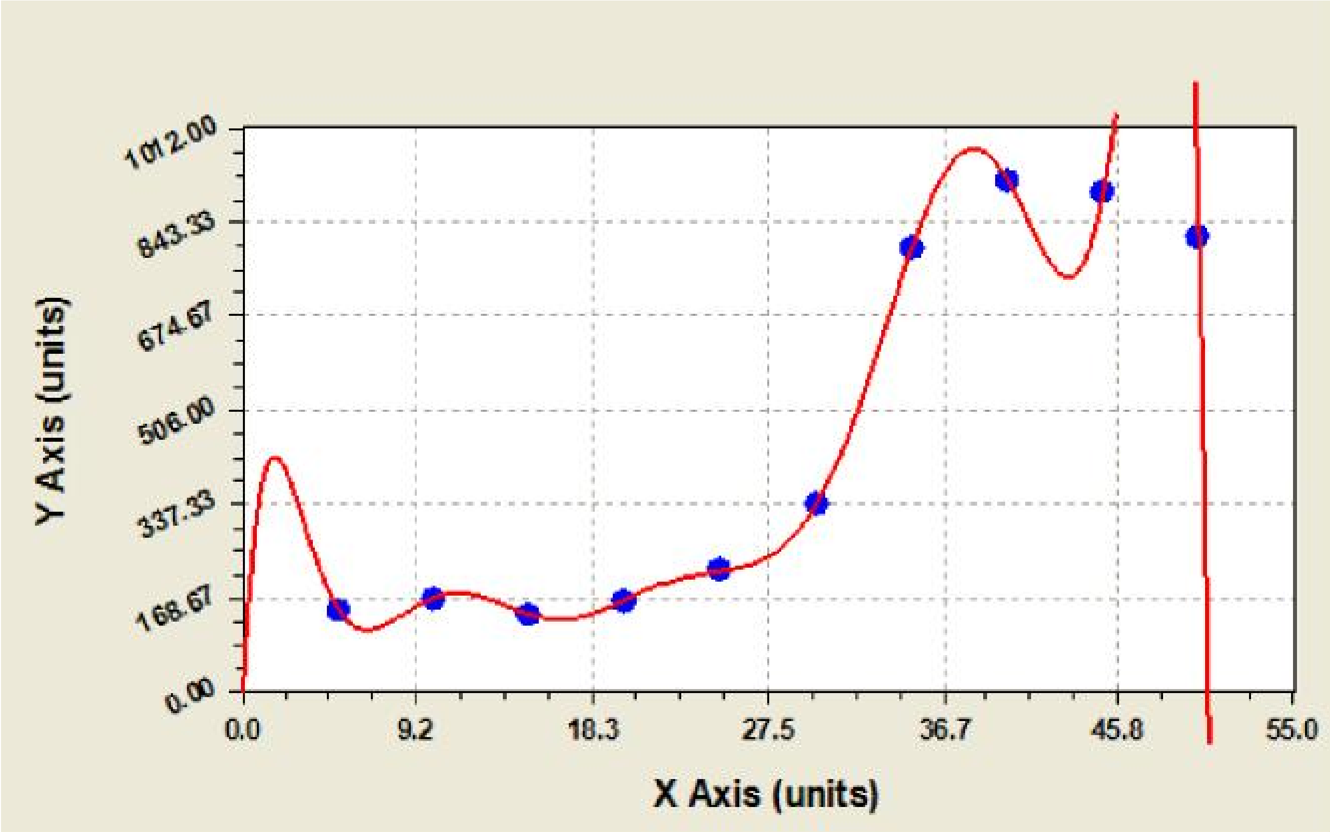


Fig 4 . *Graph of the Lagrangian Interpolation fit .*

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